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MA. CRISTELLE A. SAN ANTONIO

CN01
CN30



Revised

AUG. 11, 2008
HW/SW/Q

III- CS

2a.) FIND AN EQUATION OF THE CIRCLE THAT PASSES THROUGH THE POINTS

a.) ^A(4, 6) ^B(-6, 2) ^C(1, -3)

— • ^A(4, 6)

—

—

^B(-6, 2) •

| | | | | + | | | | |

—

—

—

• ^C(1, -3)

My

EQUATION OF LINE OF POINTS A & B

$$\rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\rightarrow y - k = \frac{q - k}{p - h} (x - h)$$

SO, LET (h, k) & (p, q)
4, 6 -6, 2

$$y - 6 = \frac{2 - 6}{-6 - 4} (x - 4)$$

$$y - 6 = \frac{2}{5} (x - 4)$$

$$y = \frac{2}{5}x - \frac{8}{5} + 6$$

$$y = \frac{2}{5}x + \frac{22}{5}$$

$$y = 0.4x + 4.4$$

EQUATION OF LINE OF POINTS A & C

LET (h, k) & (p, q)
4, 6 1, -3

$$y - 6 = \frac{-3 - 6}{1 - 4} (x - 4)$$

$$y - 6 = 3(x - 4)$$

$$y = 3x - 6$$

MIDPOINT OF LINE AB

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\left(\frac{h+p}{2}, \frac{k+q}{2} \right)$$

$$\left(\frac{4+6}{2}, \frac{6+2}{2} \right)$$

$$(-1, 4)$$

MIDPOINT OF LINE AC

$$\left(\frac{4+1}{2}, \frac{6+3}{2} \right)$$

$$\left(\frac{5}{2}, \frac{3}{2} \right)$$

$$(2.5, 1.5)$$

LINE \perp TO \overline{AB}

$$y = \frac{2}{5}x + \frac{22}{5}$$

$$y - 4 = -\frac{5}{2}(x + 1)$$

$$y = -\frac{5}{2}x + -\frac{5}{2} + 4$$

$$y = -\frac{5}{2}x + \frac{3}{2}$$

LINE \perp TO AC

$$y = 3x - 6$$

$$y - \frac{3}{2} = -\frac{1}{3}\left(x - \frac{5}{2}\right)$$

$$y = -\frac{1}{3}x + \frac{5}{6} + \frac{3}{2}$$

$$y = -\frac{1}{3}x + \frac{7}{3}$$

$l_{\perp AB} \cap l_{\perp AC}$

$$y = -\frac{5}{2}x + \frac{3}{2}$$

$$y = \frac{1}{3}x - \frac{7}{3}$$

$$0 = -\frac{13}{6}x - \frac{5}{6}$$

$$+\frac{5}{6} = -\frac{13}{6}x$$

$$x = -\frac{5}{13}$$

$$x = -\frac{2}{5}y + \frac{3}{5}$$

$$-x = 3y - 7$$

$$0 = \frac{13}{5}y - \frac{32}{5}$$

$$\frac{32}{5} = \frac{13}{5}y$$

$$y = \frac{32}{13}$$

CIRCUMCENTER $\left(\frac{h}{13}, \frac{k}{13} \right)$

POINT A (4, 6)
x y

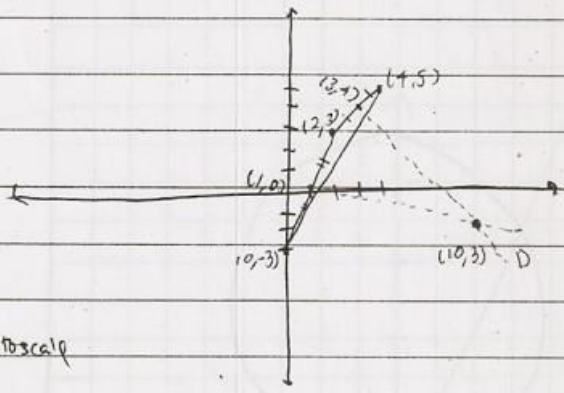
$$\left(4x + \frac{5}{13} \right)^2 + \left(6y - \frac{32}{13} \right)^2 = r^2$$

$$r^2 = \frac{5365}{169} \quad ; \quad r = 5.634320629$$

$$\left(x + \frac{5}{13} \right)^2 + \left(y - \frac{32}{13} \right)^2 = \frac{5365}{169}$$

10
10
A

7b. Passes through points (2,3), (4,5) and (0,-3)



$$AB = \sqrt{(0-2)^2 + (-3-3)^2}$$

$$= \sqrt{4+36}$$

$$= \sqrt{40} = 2\sqrt{10}$$

$$BC = \sqrt{(2-4)^2 + (3-5)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

BD = 10
 Eq n of L →

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

$$y - 3 = \frac{(-3-3)}{(0-2)} (x-2)$$

$$y - 3 = \frac{-6}{-2} (x-2)$$

$$y - 3 = 3x - 6$$

$$y = 3x - 3$$

$$M: \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M'_{AB}: \left(\frac{2+0}{2}, \frac{3+(-3)}{2} \right)$$

$$M: = \left(\frac{1, 0}{x_1 \quad y_1} \right)$$

$$y - 3 = \frac{(5-3)}{(4-2)} (x-3)$$

$$y - 3 = x - 2$$

$$y = x + 1$$

$$M_{BC}: \left(\frac{4+2}{2}, \frac{5+3}{2} \right)$$

$$M_{BC}: (3, 4)$$

AB $y - 0 = -1/3 (x - 1)$

$$y = -1/3x + 1/3$$

BC $y - 4 = -1(x - 3)$

$$y = -x + 7$$

$$-1/3x + 1/3 = -x + 7$$

$$2/3x = 20/3$$

$$x = 10$$

$$y = -3$$

$$AD = \sqrt{(10-2)^2 + (-3-3)^2}$$

$$= \sqrt{100}$$

$$(x-10)^2 + (y+3)^2 = 100$$

Arcega, Jose Mari
Perlada, Micaela

III - Cesium

$$8) x^2 + y^2 = 5y, \quad (-2, 1)$$

$$x^2 + y^2 - 5y = 0$$

$$x^2 + y^2 + 5y + \frac{25}{4} = \frac{25}{4}$$

$$x^2 + (y - \frac{5}{2})^2 = \frac{25}{4}$$

\therefore center $(0, \frac{5}{2})$

* get the slope

$$m = \frac{\frac{5}{2} - 1}{0 - (-2)} = \frac{3}{4}$$

$$y = \frac{3}{4}x + \frac{5}{2}$$

* get the \perp

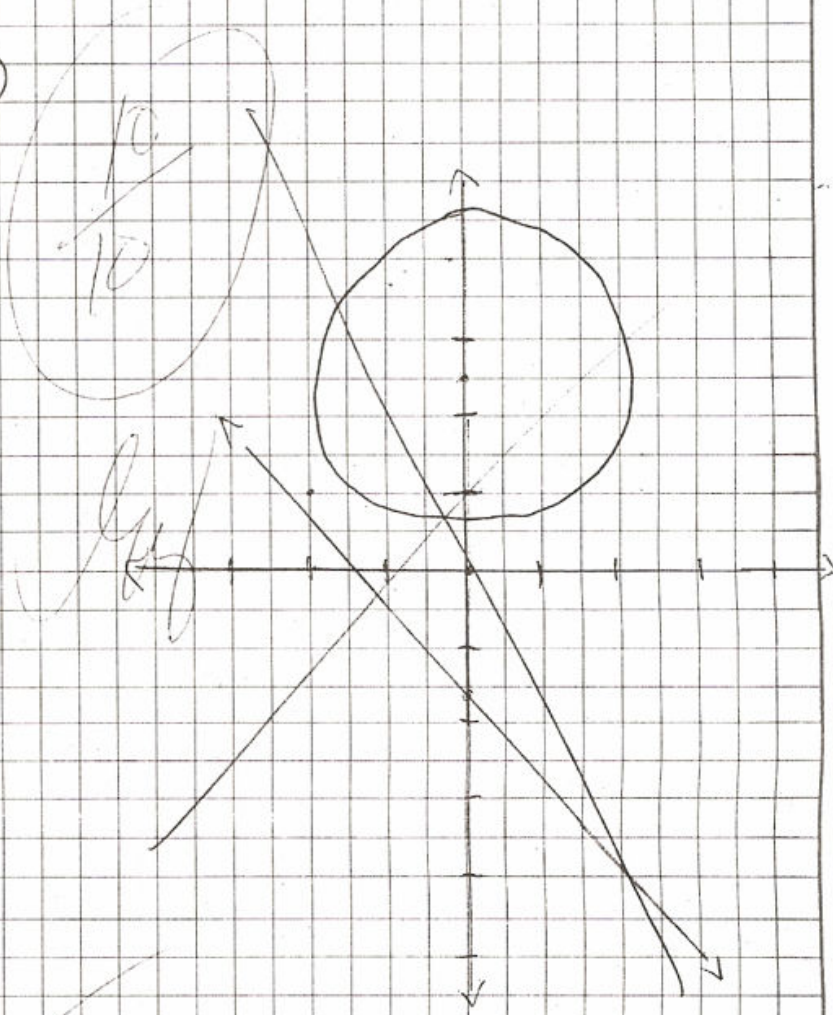
$$y = -\frac{4}{3}x + b$$

$$y - y_1 = m(x - x_1)$$

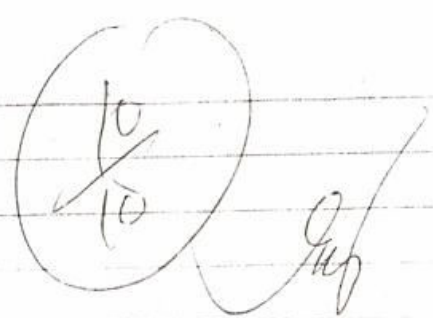
$$1 = \frac{8}{3} + b$$

$$b = -\frac{5}{3}$$

$$y = -\frac{4}{3}x - \frac{5}{3}$$



Joannes Luke Acis
Manjaira Padlan



III. Kesimpulan

Given #9

Find the equation of the circle including the circle $x^2 + y^2 + 2x - 6y + 5 = 0$ at $(1, 2)$ and passing through $(4, 1)$

Circle 1: $x^2 + y^2 + 2x - 6y + 5 = 0$
 $(x^2 + 2x + 1) + (y^2 - 6y + 9) = 5$
 $(x+1)^2 + (y-3)^2 = 5$
 C: $(-1, 3)$
 radius: $\sqrt{5}$

Circle 2,

• first equation

$$(1-h)^2 + (2-k)^2 = r^2$$

• second eq'n $(4-h)^2 + (1-k)^2 = r^2$

$$(1-h)^2 + (2-k)^2 = (4-h)^2 + (1-k)^2$$

$$1 - 2h + h^2 + 4 - 4k + k^2 = 16 - 8h + h^2 + 1 - 2k + k^2$$

$$6h - 2k = 12$$

$$\boxed{3h - k = 6}$$

• theorem:

$$m = \frac{2-3}{1+1} = \frac{-1}{2}$$

$$\frac{-1}{2} = \frac{k-2}{h-1}$$

$$-h+1 = 2k-4$$

$$-h-2k = -5$$

Third eq'n $\boxed{h = 5 - 2k}$

Substitute.

$$3(5-2k) - k = 6$$

$$15 - 6k - k = 6$$

$$15 - 7k = 6$$

$$15 - 6 = 7k$$

$$9 = 7k$$

$$\boxed{\frac{9}{7} = k}$$

$$3\left(\frac{9}{7}\right) - 2k = 6$$

$$3h = \frac{42}{7} + \frac{9}{7}$$

$$3h = \frac{51}{7}$$

$$h = \frac{17}{7}$$

$$\boxed{h = \frac{17}{7}}$$

$$\left(x - \frac{17}{7}\right)^2 + \left(y - \frac{9}{7}\right)^2 = r^2$$

radius.

$$\left(1 - \frac{17}{7}\right)^2 + \left(2 - \frac{9}{7}\right)^2 = r^2$$

$$\frac{100}{49} + \frac{25}{49} = r^2$$

$$\left(\frac{7}{7} - \frac{17}{7}\right)^2 + \left(\frac{14}{7} - \frac{9}{7}\right)^2 = r^2$$

$$\frac{125}{49} = r^2$$

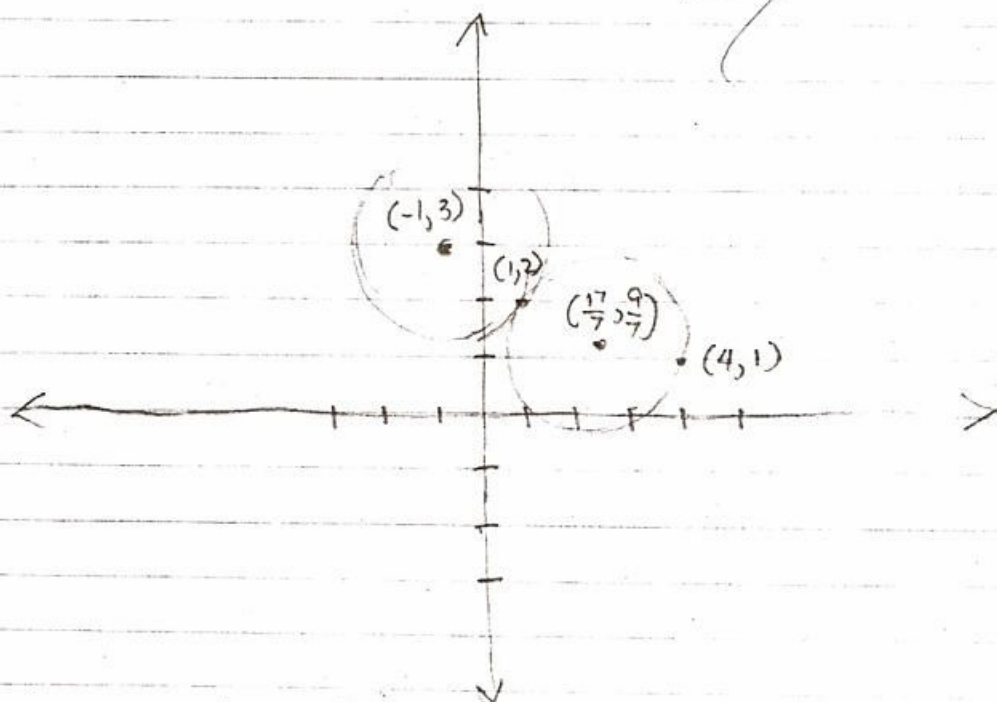
$$\left(-\frac{10}{7}\right)^2 + \left(\frac{5}{7}\right)^2 = r^2$$

Marylaine P. Padlan
Joannes Luke B. Aras
III - Cesium

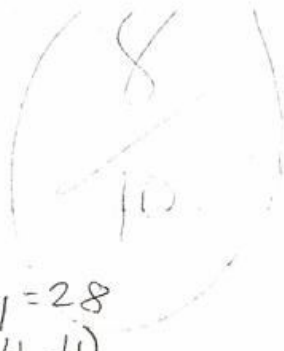
Problem # 9

$$(x+1)^2 + (y-3)^2 = 5$$

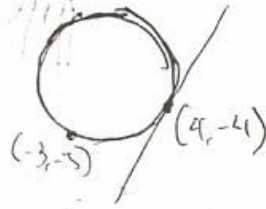
$$\left(x - \frac{17}{7} \right)^2 + \left(y - \frac{9}{7} \right)^2 = \frac{125}{49}$$



Cesium 26 and 5
Mendoza and Astrologo



10) Given: Eq'n of line: $4x - 3y = 28$
point of tangency: $(4, -4)$
point in the circle: $(-3, -5)$



First: Eq'n of Circle

1) $4x - 3y = 28$, p.t. $(4, -4)$, $(-3, -5)$
perpendicular to $4x - 3y = 28$:

$$3y = 4x - 28$$

$$y = \frac{4}{3}x - \frac{28}{3} \rightarrow \text{slope} = -\frac{3}{4}, \text{ p.t. } 4, -4$$

$$y + 4 = -\frac{3}{4}(x - 4)$$

$$y + 4 = -\frac{3}{4}x + 3$$

$$y = -\frac{3}{4}x - 1$$

radius:

$$(0 - 4)^2 + (-1 + 4)^2 = d^2$$

$$(-4)^2 + (-3)^2 = d^2$$

$$16 + 9 = d^2$$

$$25 = d^2$$

Eq'n of circle:

$$(x - 0)^2 + (y + 1)^2 = 25$$

2) Eq'n of chord p.t. $(4, -4)$ and $(-3, -5)$

$$y + 4 = \frac{-5 + 4}{-3 - 4}(x - 4)$$

$$y + 4 = \frac{-1}{-7}(x - 4)$$

$$y + 4 = \frac{1}{7}x - \frac{4}{7}$$

$$y = \frac{1}{7}x - \frac{32}{7}$$

Eq'n of line \perp to $y = \frac{1}{7}x - \frac{32}{7}$

$$(\Delta x, \Delta y) = \left(\frac{4 - 2}{2}, \frac{-4 - 5}{2}\right) \text{ or } \left(\frac{1}{2}, -\frac{9}{2}\right)$$

$$y + \frac{9}{2} = -7\left(x - \frac{1}{2}\right)$$

$$y + \frac{9}{2} = -7x + \frac{7}{2}$$

$$y = -7x - 1$$

$$\rightarrow y = -\frac{3}{4}x \cap y = -7x$$

$$x = 0, y = (0) - 1$$

$$y = -1 \rightarrow C(0, -1)$$

Lim, Rachelle A.

Co Ting Kett, Lance



11. Find the equation of the tangent line to $x^2 + y^2 - 2x - 8y + 15 = 0$ and parallel to $x - y = 9$

eq. of the circle: $x^2 + y^2 - 2x - 8y + 15 = 0$

circle: $(x^2 - 2x + 1) + (y^2 - 8y + 16) = -15 + 16 + 1$

$$(x-1)^2 + (y-4)^2 = 2$$

$$C: (1, 4), r = \sqrt{2}$$

eq. of the // line: $y = x - 9$, slope: 1

eq. of the line \perp

to the // line passing through the center

$$y - 4 = -1(x - 1)$$

$$y - 4 = -x + 1$$

$$y = -x + 5$$

pts of tangency

tangency

$$y = -x + 5 \cap (x-1)^2 + (y-4)^2 = 2$$

$$x = -y + 5$$

$$(-y + 5 - 1)^2 + (y - 4)^2 = 2$$

$$(-y + 4)^2 + (y - 4)^2 = 2$$

$$(y^2 - 8y + 16) + (y^2 - 8y + 16) = 2$$

$$2y^2 - 16y + 32 = 2$$

$$y^2 - 8y + 15 = 0$$

$$(y-3)(y-5) = 0; y = 3 \text{ or } y = 5$$

$$y_1 = -x_1 + 5$$

$$y_2 = -x_2 + 5$$

$$3 = -x_1 + 5$$

$$5 = -x_2 + 5$$

$$x_1 = 2$$

$$x_2 = 0$$

$$(2, 3)$$

$$(0, 5)$$

• equations of the tangent lines

$$y - 3 = (x - 2)$$

$$y = x - 2 + 3$$

$$\boxed{y = x + 1}$$

$$y - 5 = (x - 0)$$

$$\boxed{y = x + 5}$$

Bea Clarise Garcia
 Llogene de Loyola

III - Cesium
 CN: 7, 24

Given: $(-4, -2); (2, 0)$
 $5x - 2y = 19 \iff y = \frac{5}{2}x - \frac{19}{2}$

$d^2 = (x - x_1)^2 + (y - y_1)^2$
 $(-4, -2); (x, y)$
 $(2, 0); (x, y)$

$(x + 4)^2 + (y + 2)^2 =$
 $(x - 2)^2 + y^2$
 $x^2 + 8x + 16 + y^2 + 4y + 4 =$
 $x^2 - 4x + 4 + y^2$

$8x + 16 + 4y + 4 = -4x + 4$

$12x + 4y + 16 = 0$
 $(5x - 2y = 19) \cdot 2$

$12x + 4y + 16 = 0$
 $10x - 2y - 38 = 0$

$22x = 22$

$x = 1$

$y = \frac{5}{2}(1) - \frac{19}{2}$

$y = \frac{-14}{2} \iff -7$

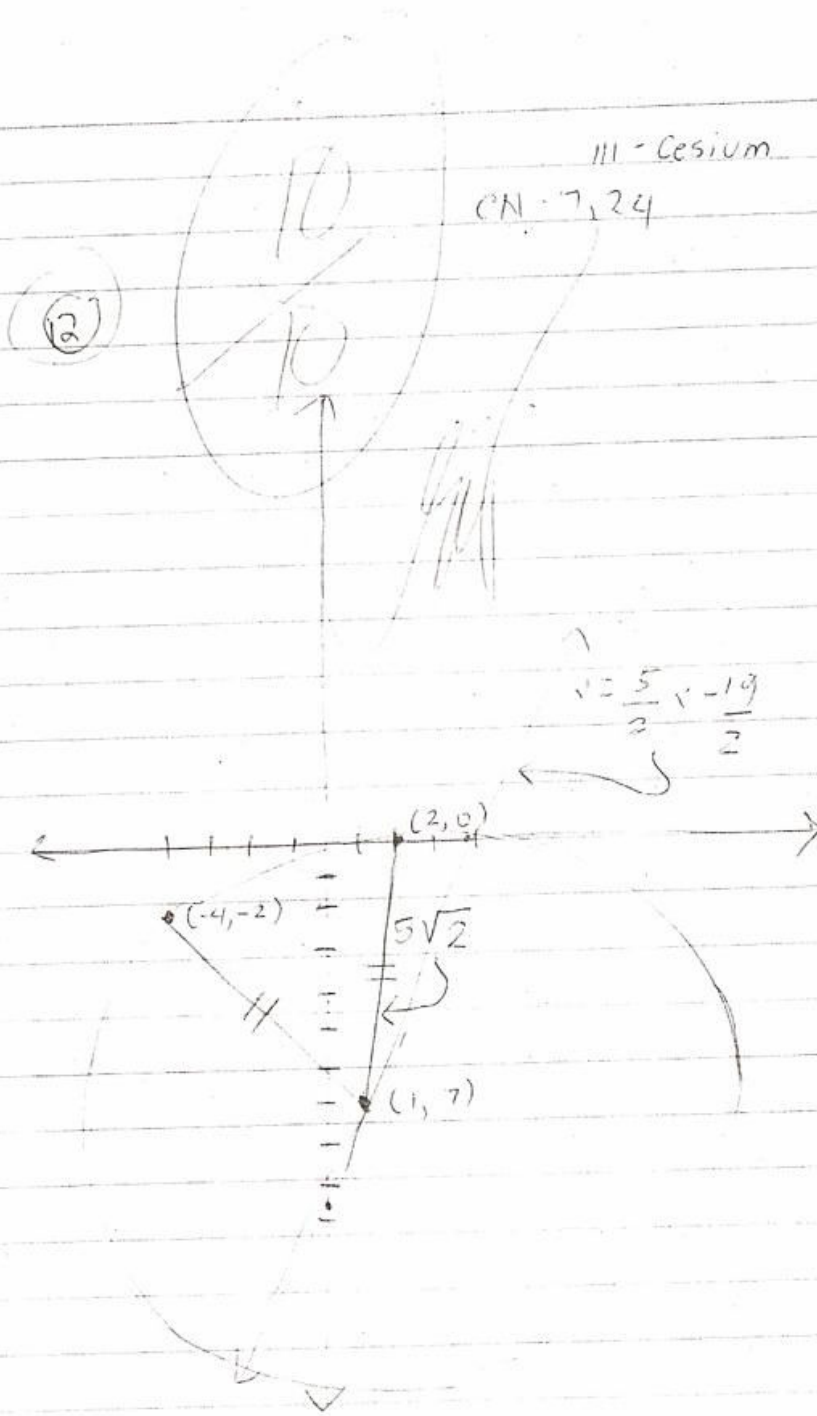
$d = \sqrt{(2 - 1)^2 + (0 + 7)^2}$

$d = \sqrt{1 + 49}$

$d = \sqrt{50} \iff |5\sqrt{2}| = r$

EQN =

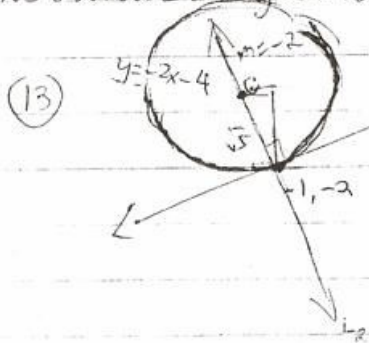
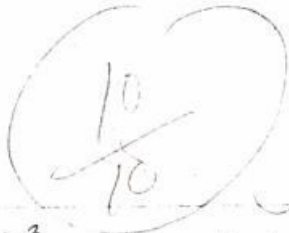
$(x - 1)^2 + (y + 7)^2 = 50$



B

III - Cesium
 8 Jash Dizon
 23 Cara Evangelista

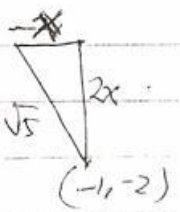
8 August 2008
 Fairwork
 Given $x - 2y = 3$
 $r = \sqrt{5}$
 $P(-1, -2)$



Since $L_1 \perp L_2$, $m_2 = -2$
 $L_2 = y + 2 = -2(x + 1)$
 $y = -2x - 4$

From $y = -2x - 4$
 we get for every 2y or x change.

$$m_2 = \frac{-2}{1} = \frac{\Delta y}{\Delta x} = \frac{2x}{-x} = \frac{-2x}{x}$$



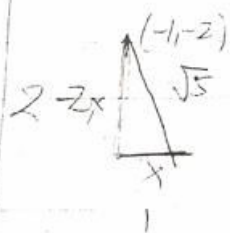
$$x^2 + 4x^2 = (\sqrt{5})^2$$

$$5x^2 - 5 = 0$$

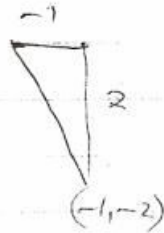
$$(x+1)(x-1) = 0$$

$$x = -1, 1$$

$$\underline{\underline{x = 1}}$$



$x = 1$



So we get coordinates of center:
 $C_1(-2, 0)$ and $C_2(0, -4)$

\therefore S.E. of the circle is

$$(x+2)^2 + y^2 = 5$$

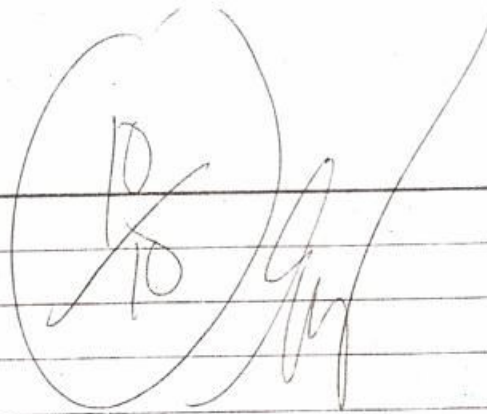
and

$$x^2 + (y+4)^2 = 5$$

Justin Dumayag #9

Michelle Co #22

~~Justin~~
Justin



III - Cesium

14. Find the equation of the circle passing through the point $(7, 9)$, tangent to the x-axis and has its center on the line $x - y + 1 = 0$

given: point $(7, 9)$

center on $y = x + 1$ $C(x, x + 1)$

point tangent to x-axis $(x, 0)$

$$(7 - x)^2 + (9 - (x + 1))^2 = r^2 \quad [\text{where } r = x + 1 \text{ because } x + 1 \text{ is the distance from the center to the point tangent to the x-axis}]$$

$$2x^2 - 30x + 149 = x^2 + 2x + 1$$

$$2x^2 - 30x + 148 = x^2 + 2x + 1$$

$$x^2 - 32x + 147 = 0$$

$$(x - 4)(x - 28) = 0$$

\therefore There are two centers which satisfy the conditions;

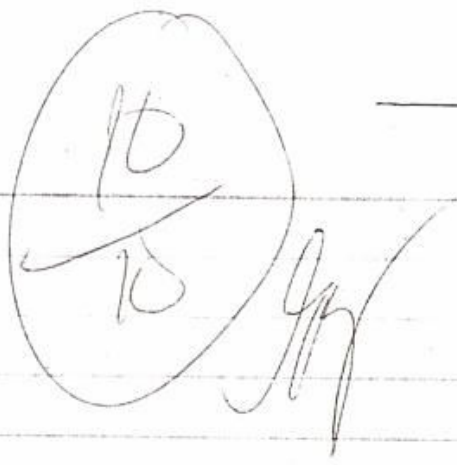
$C(4, 5)$ and $C(28, 29)$

The equations are:

$$(x - 4)^2 + (y - 5)^2 = 25$$

$$(x - 28)^2 + (y - 29)^2 = 841$$

Cs 10, 21
ENDICO, Marxen
TAN, Bryan



15.

$$2x + y + 15 = 0 \rightarrow y = -2x - 15$$

$$2x + y - 5 = 0 \rightarrow y = -2x + 5$$

to get equation of perpendicular passing through (2, 1):

$$y - 1 = \left(\frac{1}{2}\right)(x - 2)$$

negative reciprocal

$$y - 1 = \frac{1}{2}x - 1$$

$$y = \frac{1}{2}x$$

using systems of linear equations:

$$y = \frac{1}{2}x$$

$$2x + y + 15 = 0$$

$$\frac{1}{2}x - y = 0$$

$$\underline{2x + y = -15}$$

$$5/2x = -15$$

$$x = -6$$

$$y = -3$$

∴ the line $y = \frac{1}{2}x$ intersects the line $2x + y + 15 = 0$ at point $(-6, -3)$

∴ $(-6, -3)$ and $(2, 1)$ are endpoints of the circle

Using distance formula:

$$\sqrt{(-6-2)^2 + (-3-1)^2} = \frac{\sqrt{80}}{2} = 4\sqrt{5} \rightarrow \text{diameter}$$

$$\text{radius} = 2\sqrt{5}$$

To find the center:

$$x\text{-coordinate} = \frac{-6+2}{2} = -2$$

$$y\text{-coordinate} = \frac{-3+1}{2} = -1$$

$$\therefore \text{center} = (-2, -1)$$

Therefore, the equation of the circle is:

$$\text{SF: } (x+2)^2 + (y+1)^2 = 20$$

$$\text{GF: } x^2 + y^2 + 4x + 2y - 15 = 0$$



III-C5

CN II GABO, Eugene Paolo

8 August 08

20 SAN JUAN, Marck Darryl

$$16) \textcircled{1} \rightarrow x^2 + y^2 - 6x + 2y - 15 = 0$$

$$x^2 - 6x + 9$$

$$y^2 + 2y + 1$$

$$(x-3)^2 + (y+1)^2 = 25$$

$$C: (3, -1)^2 \quad r = 5$$

$$\text{radius} = \sqrt{(3 - \frac{24}{13})^2 + (-1 + \frac{24}{13})^2}$$

$$= \sqrt{1}$$

$$= 1$$

$$\textcircled{2} \rightarrow (x-3)^2 + (y+1)^2 = 1$$

Tangent line $5x + 12y + 10 = 0$

(L1)

$$12y = -5x - 10$$

$$y = \frac{-5x - 10}{12}$$

$$m = \frac{-5}{12}$$

$$\therefore m_2 = \frac{12}{5}$$

$$L_2 \quad y + 1 = \frac{12}{5}(x - 3)$$

$$5y + 5 = 12x - 36$$

$$x12 \begin{cases} 5x + 12y = -10 & L_1 \\ -12x + 5y = -41 & L_2 \end{cases}$$

$$x5 \begin{cases} 5x + 12y = -10 & L_1 \\ -12x + 5y = -41 & L_2 \end{cases}$$

$$60x + 144y = -120$$

$$-60x + 25y = -205$$

$$169y = -325$$

$$y = \frac{-25}{13}$$

$$(\text{substitute}) x = \frac{34}{13}$$

CS 12, 19

GARCIA, James Rainier M.

PEREZ, Jose Nichola

15 Aug 2008

2Q, Pbs

17. $r = 2\sqrt{5}$

tangent to $y = 2x$

passes through $P(3, -4)$

$(x-h)^2 + (y-k)^2 = r^2 \dots (1)$

Plug in $P(3, -4)$

$(3-h)^2 + (-4-k)^2 = 20 \dots (2)$

\perp to $y = 2x$ & passes through $P(3, -4)$

$y = 2x - 10$

$k = 2h - 10 \dots (3)$

$(2) \rightarrow (1)$

$(3-h)^2 + (-4-2h+10)^2 = 20$

$h^2 - 6h + 9 + (-2h+6)^2 = 20$

$h^2 - 6h + 9 + 4h^2 - 24h + 36 = 20$

$[5h^2 - 30h + 45 = 20] \div 5$

$h^2 - 6h + 9 = 4$

$h^2 - 6h + 5 = 0$

$(h-5)(h-1) = 0$

$h = 5, 1 \dots (4)$

$(4) \rightarrow (3)$

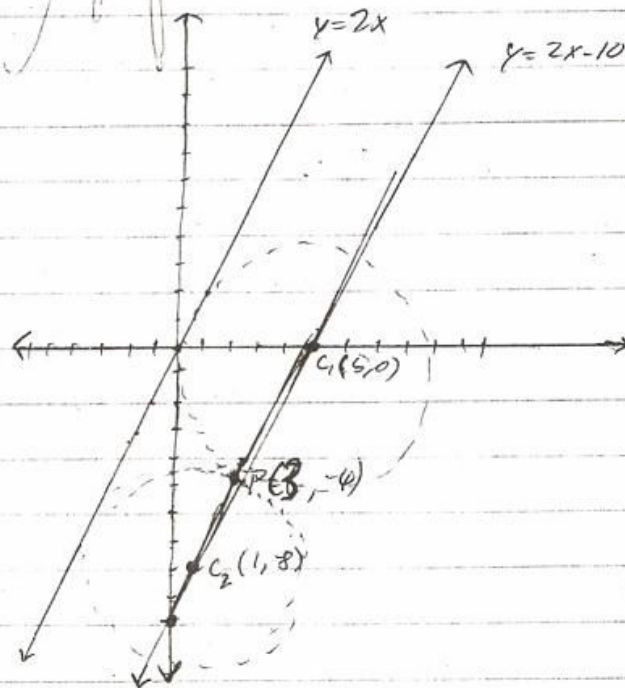
$k = 2(5) - 10 \cup k = 2(1) - 10$

$= 10 - 10$

$= 2 - 10$

$k = 0$

$k = -8 \dots (5)$



$(4) \rightarrow (1) \cup (5) \rightarrow (1)$
 $(x-5)^2 + (y-0)^2 = 20$ or $(x-1)^2 + (y+8)^2 = 20$

The circle may be: $(x-5)^2 + y^2 = 20$

or $(x-1)^2 + (y+8)^2 = 20$

18

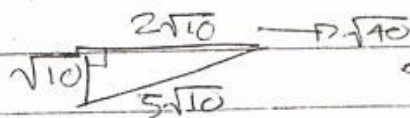
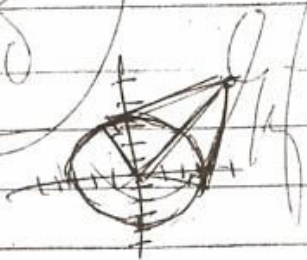
Palnicki Ger, Jeff Poscasco

8/11/2008

III - Cs

#18 $x^2 + y^2 = 10$

P(5,5)



$$(x-5)^2 + (y-5)^2 = 10$$

$$x^2 - 10x + y^2 - 10y = -10$$

$$-10x - 10y = -20$$

$$x + y = 2$$

$$x = 2 - y$$

$$(2-y)^2 + y^2 = 10$$

$$y^2 - 4y + 4 + y^2 = 10$$

$$2y^2 - 4y - 6 = 0$$

$$y^2 - 2y - 3 = 0$$

$$(y-3)(y+1) = 0$$

P1's (-1,3) \rightarrow P(5,5)(-1,3)

(3,-1)

$$y-5 = \frac{3-5}{-1-5}(x-5)$$

$$y-5 = -\frac{2}{6}(x-5)$$

(5,5)(3,-1)

$$y-5 = \frac{5-(-1)}{5-3}(x-5)$$

$$y-5 = \frac{1}{3}x - \frac{5}{3}$$

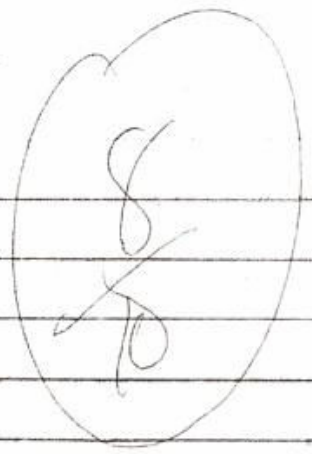
$$y-5 = 3(x-5)$$

$$y-5 = 3x-15$$

$$y = 3x-10$$

$$y = \frac{1}{3}x + \frac{10}{3}$$

Rabee Nunes - 17
Andrei Macandog - 14



3- Cs

Points $(1, -4)$, $(3, -2)$, $(5, 2)$

circumscribed by a circle

$(1, -4)$ $(3, -2)$

midp. = $(2, -1)$

eq. \perp line is $y = -x + 1$
 $(1, -4)$ $(3, -2)$

midpoint = $(1, -3)$

eq. \perp line is $y = 2x - 1$

$$2x - 1 = -x + 1$$

$$2x = -x + 2$$

$$-x = 2$$

$$x = -2$$

~~$2(-2) - 1 = y$~~ $y = -(-2) + 1$

~~$-4 - 1 = y$~~ $y = 3$

C of C = $(2, 3)$

$$r = \sqrt{(1+2)^2 + (-4-3)^2}$$

$$r^2 = \sqrt{50} = 5\sqrt{2}$$

$$50 = (-1+2)^2 + (-4-3)^2$$



X

4

Date: _____

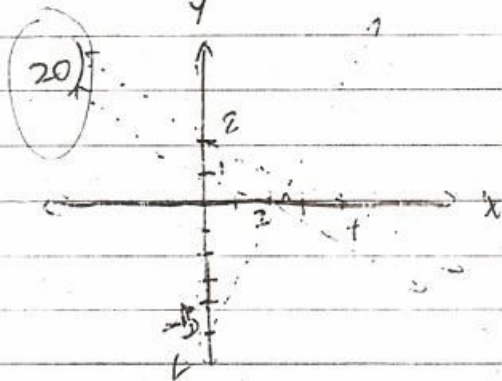
Nogales, Ernest Nathan, L.

Ng, Joe I
III - Cesium

to
10

perpendicular

Aug. 8, 2008



To get the radius, find half the distance between the 2 parallel lines $x+2y=4$ and $x+2y=2$. Since $y=2x-5$ is perpendicular to both, we can find the points of intersection between $y=2x-5$ and $x+2y=4$, and $y=2x-5$ and $x+2y=2$.

There are 2 possible circles.

 $L_1 \cap L_3$

$$x+2y=4 \cap y=2x-5$$

$$\begin{cases} y=2x-5 \\ y=-\frac{x}{2}+2 \end{cases}$$

$$2x-5 = -\frac{1}{2}x+2$$

$$\frac{5}{2}x = 7$$

$$x = \frac{14}{5}$$

$$y = \frac{3}{5}$$

$$P\left(\frac{14}{5}, \frac{3}{5}\right)$$

 $L_2 \cap L_3$

$$x+2y=2 \cap y=2x-5$$

$$\begin{cases} y=2x-5 \\ y=-\frac{x}{2}+1 \end{cases}$$

$$2x-5 = -\frac{x}{2}+1$$

$$\frac{5}{2}x = 6$$

$$x = \frac{12}{5}$$

$$y = -\frac{1}{5}$$

$$Q\left(\frac{12}{5}, -\frac{1}{5}\right)$$

The diameter of the circle is the distance between P and Q

$$\sqrt{\left(\frac{14}{5} - \frac{12}{5}\right)^2 + \left(\frac{3}{5} - \frac{-1}{5}\right)^2} = \frac{2\sqrt{5}}{5}$$

The radius is $\frac{\sqrt{5}}{5}$.

The line L to $y = 2x - 5$ and passing through the midpoint of P and Q also passes through the center of the circle.

midpoint of P and Q is R

$$R\left(\frac{14}{5}, \frac{1}{5}\right)$$

$$L \text{ to } y = 2x - 5 \Rightarrow y = -\frac{x}{2} + \frac{3}{2}$$

The circle with center R and radius $\frac{\sqrt{5}}{5}$ also passes through the center of the circles that are tangent to L_1 , L_2 , and L_3 .

$$\left\{ \begin{aligned} \left(x - \frac{13}{5}\right)^2 + \left(y - \frac{1}{5}\right)^2 &= \frac{1}{5} \\ y &= -\frac{1}{2}x + \frac{3}{2} \end{aligned} \right.$$

$$\left(x - \frac{13}{5}\right)^2 + \left(-\frac{1}{2}x + \frac{13}{10}\right)^2 = \frac{1}{5}$$

$$x^2 - \frac{26x}{5} + \frac{169}{25} + \frac{1}{4}x^2 - \frac{13x}{10} + \frac{169}{100} = \frac{1}{5}$$

$$\frac{5}{4}x^2 - \frac{13}{2}x + \frac{33}{4} = 0$$

$$5x^2 - 26x + 33 = 0$$

$$(5x - 11)(x - 3) = 0$$

$$x = 3 \text{ or } x = \frac{11}{5}$$

$$y = 0 \text{ or } y = \frac{2}{5}$$

The equation of circles tangent to L_1 , L_2 and L_3 are $(x-3)^2 + y^2 = \frac{1}{5}$ and $\left(x - \frac{11}{5}\right)^2 + \left(y - \frac{2}{5}\right)^2 = \frac{1}{5}$.